NATIONAL UNIVERSITY OF ENGINEERING
COLLEGE OF SCIENCES
MATHEMATICS PROGRAM

CM411 – MEASUREMENT THEORY

I. GENERAL INFORMATION

CODE	: CM411 Measurement Theory
SEMESTER	: 7
CREDITS	: 5
HOURS PER WEEK	: 6 (Theory – Practice)
PREREQUISITES	: CM312 Analysis of Complex-Valued Functions CM394 Multivariable Real Functions II
CONDITION	: Mandatory

II. COURSE DESCRIPTION

To describe and recognize the measurable sets, measurable functions and the Lebesgue integral. Interpret the Lebesgue derivation and integral.

III. LEARNING UNITS

1. Preliminaries in the Theory of Sets

Elementary Theory of Sets / Family of Sets / Morgan's Laws / Relationships between sets / Cartesian Product / Binary Relations and Functions / Partial-Order Relations, Equivalence Relations / General Cartesian Product. The Axiom of Choice / The Axiom of Choice and some of its equivalences (Tukey's lemma, Hausdorff's principle of maximality, Zorn's lemma, Good order principle) / Cardinal Numbers / Countable and uncountable sets / Equipotent sets, Notion of cardinal number / Finite sets, countable and uncountable / Schröeder-Bernstein’s Theorem. Cantor's Theorem / Tarski and Dedekin's Theorem. Ring, algebra, semi-ring, semi-algebra, σ - ring, σ - set algebra / Algebra generated by a family of sets / Borel sets.

2. Measurements and Measurable Sets

Measurable spaces and construction of measures / Measures in semi-rings. Notion of measurable sets / Extension of a measure in a semi-ring to a ring / Hereditary ring / External measurement / Extension of a measurement in a ring to an external measurement in a hereditary ring / Caratheodory Extension Theorem / Hahn Extension Theorem (Existence and uniqueness of a complete measure) / Lebesgue measure / The Idea of measurement, length, area and volume. Definition of the Lebesgue measure / Lebesgue-measurable sets / Approximation of Lebesgue-measurable sets by open sets, closed sets, compact R^n sets. Non-Lebesgue-measurable and non-Borelian sets.

3. **The Lebesgue Integral**

Measurable functions / Topology of extended reals / Measurable functions (at extended real values) / Positive part and negative part of a function / Special measurable functions / Simple function, step function, characteristic function / Operations and fundamental properties of measurable functions / Approximation of functions measurable by simple function sequences / Convergence in measure / Riesz's Theorem / Egoroff's Theorem / Lebesgue Abstract Integral / Introduction / Riemman's Integral. Integral of a non-negative measurable function / Fundamental properties / Lebesgue's monotone convergence theorem / Fatou's lemma. Integral of a real (extended) and complex function / Fundamental properties / Lebesgue's dominated convergence theorem / Riemman Integral and Lebesgue Integral.

4. **Derivation and Integration**

Introduction / Initial concepts / Lower and upper lateral derivatives / Monotonic functions and fundamental properties / Vitali's cover. Vitali's lemma / Differentiability of a monotonous function (Lebesgue's Theorem) / First Fundamental Theorem / Bounded variation functions / Variation of a function / Characterization of bounded variation functions. Absolutely continuous functions / Characterization of absolutely continuous functions / Second fundamental theorem**.**

IV. BIBLIOGRAPHY

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- Dugundji, J., Topology, Boston, Allyn and Bacon 1966.
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- Kelley, J.L., General Toplogy, New York, Van Nostrand 1955.
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- Lima, E.L., Curso de Analise (Vol. 1), Rio de Janeiro, IMPA.
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