



**NATIONAL UNIVERSITY OF ENGINEERING**  
**COLLEGE OF SCIENCES**  
**MATHEMATICS PROGRAM**

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**CM214 – REAL VARIABLE FUNCTIONS**

**I. GENERAL INFORMATION**

<b>CODE</b>	: CM214 Real Variable Functions
<b>SEMESTER</b>	: 4
<b>CREDITS</b>	: 7
<b>HOURS PER WEEK</b>	: 9 (Theory – Practice)
<b>PREREQUISITES</b>	: CM211 Differential Calculus and Advanced Integral
<b>CONDITION</b>	: Mandatory

**II. COURSE DESCRIPTION**

Establish the basic tools of mathematical analysis. Introduce the student in the study of successions and series of both real numbers and functions, likewise establish the basic notions of linear topology.

**III. LEARNING UNITS**

**1. Set of Real Numbers**

Peano's Axioms / First principle of mathematical induction / Second principle of mathematical induction, good order principle / Properties of natural numbers / Finite and infinite sets / Enumerable sets / Definition of the set of real numbers  $\mathbf{R}$  / Properties of  $\mathbf{R}$  / Supreme axiom / Property of embedded intervals / Arquimedian Principle / Density of  $\mathbf{Q}$ .

**2. Successions and Series of Real Numbers**

Definition of succession / Bounded succession / Limit of a succession / Limits and inequalities, limit algebra / Infinite limits / Cauchy successions / Upper limit and lower limit / Convergent series / Algebra of series / Comparison criteria and Cauchy criteria / Absolutely convergent series / Reason, root and Dirichlet criterion / Alternating series and conditionally convergent series.

**3. Topology in  $\mathbf{R}$**

Open sets / Adhesion points / Closed sets / Accumulation points / Compact sets.

**4. Function limits**

Introduction and basic notions / Functions limits / Limits algebra / Lateral limits / Limits to infinity, infinite limits / Adhesion values of a function / Upper limit and lower limit.

## **5. Continuous functions**

Continuous functions in a range / Continuous functions in compact sets / Uniform continuity.

## **6. Derivatives**

Definition and properties of the derivative in a point / Derivation rules. Taylor formula / Derivable functions in an interval / Taylor series. Analytical functions

## **7. Riemman Integral**

Superior integral and inferior integral / Integral functions / Fundamental theorem of the calculation / Integrals as a limit of sums. Characterization of the integrable functions.

## **8. Sequences and series of functions**

Punctual and uniform convergence of a succession of functions / Integration and derivation of terms / Function Series: Uniform Convergence, Weierstrass criterion. Power series, Interval and convergences radius / Derivation of power series / Defined functions by power series / Product of power series. Equicontinuity.

## **IV. BIBLIOGRAPHY**

- Apostol, T., Análisis Matemático: Introducción moderna al cálculo superior, Reverté, 1960.
- Robert Bartle, Introducción al Análisis Matemático de una Variable, Limusa.
- Lages Lima, E., Curso de Análise, Vol. I, Proyecto Euclides.
- Rudin, Walter, Principio de Análisis Matemático.
- H. L. Royden, Real Analysis. Mc Millan, 1968.