



**NATIONAL UNIVERSITY OF ENGINEERING**  
**COLLEGE OF SCIENCES**  
**PHYSICS PROGRAM**

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**CM211 – ADVANCED DIFFERENTIAL AND INTEGRAL CALCULUS**

**I. GENERAL INFORMATION**

<b>CODE</b>	: CM211 Advanced Differential and Integral Calculus
<b>SEMESTER</b>	: 3
<b>CREDITS</b>	: 7
<b>HOURS PER WEEK</b>	: 9 (Theory – Practice)
<b>PREREQUISITES</b>	: CM132 Integral Calculus, CM142 Vector Calculus II
<b>CONDITION</b>	: Compulsory

**II. COURSE DESCRIPTION**

The course prepares students in the understanding and application of multi-dimensional calculus for analyzing and solving engineering problems with application of Physics, Geometry and Optimization. The concepts and methods of vector functions, spatial curves, function of several variables, gradients, directional and partial derivatives, double and triple integrals, line and surface integrals, Gauss and Stokes theorems, and vector fields, are analyzed and applied to model and solve diverse problems. The course focus on both, clear understanding of concepts and correct application of methods for solving engineering problems.

**III. COURSE OUTCOMES**

1. Identify the scientific character of Mathematics and appraise the rigor and objectivity of the discipline.
2. Interpret the concept of vector function and apply it to calculate limits, derivatives and integrals, and appraise its importance in the solution of engineering problems.
3. Understand the concept of real functions or several variables and apply it to calculate limits, partial derivatives, and double and triple integrals.
4. Define and calculate line, rotational integral and apply Green theorem thoroughly.
5. Define and analyze rotational, divergence, surface area, Stokes and Gauss theorems, and appraise its importance in the solution of engineering problems.

**IV. COURSE CONTENTS**

**1. VECTOR FUNCTIONS**

Vector functions: limit, continuity, derivative, integral, arc length / Fundamental vectors: tangent, normal, binormal / Fundamental planes: oscillator, normal / Curvature and torsion.

**2. FUNCTIONS OF SEVERAL VARIABLES**

Real functions of several variables: limit, continuity / Partial derivative / Directional derivative, gradient, tangent plane / Chain rule / Implicit derivative / Second derivative criteria / Maximums and minimums / Lagrange multipliers for well-conditioned problems / Transformations: polar, cylindrical and spherical coordinates / Double integrals: definition and properties.

### **3. MULTIPLE INTEGRALS: DOUBLE AND TRIPLE**

Iterated double integrals, calculus of double integrals over general regions / Change of variable in double integrals / Double integrals in polar coordinates / Applications: area of plane regions, center of mass, moments of inertia / Triple integrals, geometric interpretation, volume / Change of variables in triple integrals / Triple integrals in cylindrical and spherical coordinates.

### **4. LINE INTEGRALS, STOKES AND GAUSS THEOREM**

Line integrals: definition and properties / Line integrals independent of paths / Work / Green's theorem in the plane / Area of the surface / Integral of the surface / Vector fields: rotation and divergence / The flow of vector fields / Stokes' and Gauss theorems and applications.

## **V. METHODOLOGY**

The course takes place in theory and practice sessions. In theory sessions, the instructor presents the concepts, theorems and applications. In practice sessions, different kinds of problems are modeled and solved and their solutions are analyzed. Active participation of students is encouraged in all sessions.

## **VI. GRADING SYSTEM**

The Final Grade (PF) is calculated with the following formula:

$$PF = (EP + EF + PP) / 3$$

ME: Mid-term exam

EF: Final Exam

PP: Average of quizzes

## **VII. BIBLIOGRAPHY**

- 1. STEWART, James**  
Multivariable Calculus  
I.T.E. Editions, 2010
- 2. APÓSTOL, Tom**  
Calculus, Vol. II  
Reverte Ed., 2008