



# NATIONAL UNIVERSITY OF ENGINEERING

## COLLEGE OF GEOLOGICAL, MINING AND METALLURGICAL ENGINEERING

### MINING ENGINEERING PROGRAM

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## BMA12 – VECTOR CALCULUS

### I. GENERAL INFORMATION

<b>CODE</b>	: BMA12 Vector Calculus
<b>SEMESTER</b>	: 1
<b>CREDITS</b>	: 4
<b>HOURS PER WEEK</b>	: 5 (Theory–Practice)
<b>PREREQUISITES</b>	: --
<b>CONDITION</b>	: Compulsory

### II. COURSE DESCRIPTION

The course prepares students for the analysis and calculation of matrices and their application to solve systems of linear equations. Students analyze and solve problems of analytic geometry including lines and planes in two and three dimensions, as well as conic sections such as circumferences, ellipses, parabolas and hyperbolas. The course provides students with the principles of matrices and analytic geometry to be applied to solve engineering problems.

### III. COURSE OUTCOMES

At the end of the course, students:

1. Analyze the properties of matrices and complete matrix operations including determinant and inverse.
2. Build mathematical models of matrices and system of linear equations to solve practical mining engineering problems.
3. Understand and apply analytic geometry in two and three dimensions
4. Analyze and solve problems of lines in two and three dimensions.
5. Analyze and solve problems in three dimensions.
6. Analyze and apply coordinate transformations (translation and rotation).

### IV. LEARNING UNITS

#### 1. MATRICES, DETERMINANTS AND LINEAR EQUATIONS

Matrix. Elements. Rows and columns / Order / Types of matrices. Square, null, triangular, diagonal, identity, nilpotent and periodic matrices / Equality of matrices / Operations with matrices. Addition, Multiplication of a scalar by a matrix. Product of matrices / Transpose of a matrix / Symmetric and antisymmetric matrices / Properties of determinants / Non-singular matrix / Inverse matrix / Minors and cofactors of a matrix / Adjoint matrix / Inverse matrix by the method of the adjoint / Determination of the cofactors of a matrix of order 4 / Generalization to matrices of order  $n$  / Step matrix / Calculation of the range of a matrix / Calculation of the inverse of a matrix by sequential basic operations / Solution of linear equation systems by matrix methods: Cramer's law, elementary operations.

#### 2. ANALYTIC GEOMETRY IN 2 DIMENSIONS

$R^2$ : Coordinate system in two dimensions / Distance between two points / Midpoint / Vector space in two dimensions / Radius vector and vector in  $R^2$  / Coordinates and representation of vector as oriented segment / Components of vectors in the plane / Equality of vectors, addition and multiplication of a scalar by a vector / Properties of vector operations / Cancellation between two vectors / Dot product / Properties of the dot product / Parallel vectors, geometric interpretation / Orthogonal vectors / Canonical vectors, angles and cosines / Orthogonal projection and component / Applications of vectors.

### 3. ANALYTIC GEOMETRY IN 3 DIMENSIONS

$R^3$ : Coordinate system in three dimensions / Distance between two points / Midpoint. / Vector space in three dimensions / Radio vector and vector in  $R^3$  / Coordinates and representation of the vector as oriented segment / Components of vectors in the space / Equality of vectors, addition and multiplication of a scalar by a vector / Operations with vectors / Angle between two vectors / Dot product / Parallel vectors, geometric interpretation / Orthogonal vectors / Canonical vector / Orthogonal projection and components / Vector product / Properties / Geometric interpretation / Applications / Triple dot product / Triple vector product.

### 4. LINES IN TWO AND THREE DIMENSIONS. PLANES

The line in  $R^2$ . / Tilt and slope / Relationship between slope and directional vector / General equation. Vector equation. Parametric equation / Symmetric equation (Intersection point with the 2 axis) / Angle between lines / Parallelism and orthogonality between lines / Normal equation of a straight line / Area of a triangle / Distance from a point to a line / Distance between parallel lines / Barycenter of a triangle / Lines in the space / Equation / Vector, parametric and symmetrical / Angle between two lines / Planes / Equation of the plane: Vector, parametric, normal, general and symmetric / Intersection point of the 3 axis / Distance from a point to a plane / Distance between parallel planes / Angle between planes / Intersection of planes / Angle between line and plane / Intersection of a straight line with a plane / Distance between two lines.

### 5. TRANSFORMATIONS

$R^2$  transformations / Translation and rotation of coordinate axis / Transformation of coordinates of a point and a vector in  $R^2$  / Transformations in  $R^3$  / Translation and rotation of coordinate axis. / Transformation of coordinates of a point in  $R^3$ . / Determination of the cartesian equation of a geometrical locus from given conditions / Conics / General definition / Determination of the cartesian equation of a geometrical locus from given conditions / The circumference: Definition and elements / General, cartesian and vector equation of the circumference / Tangent and normal lines to a circumference

### 6. CIRCUMFERENCE AND CONIC SECTIONS

The parabola: focus, focal axis, vertex, focal cord, straight side and vector radius / Eccentricity. / Cartesian, general, and vector equations / Tangent and normal lines to a parabola / Polar and vector forms / Applications / The ellipse: focus, focal axis, vertex, focal cord and straight side / Eccentricity / Cartesian, general and vector equations / Tangent and normal lines to an ellipse / Polar and vector representations / Applications / Hyperbola: focus, focal axis, vertex, focal cord and straight side / Eccentricity / Cartesian, general and vector equations / Tangent and normal lines to a hyperbola / Polar and vector representations.

## V. PRACTICAL EXPERIENCES:

1. Session 1: Matrices and linear equation systems.
2. Session 2: Analytic geometry in  $R^2$  and  $R^3$ .
3. Session 3: Lines in  $R^2$  and  $R^3$ .
4. Session 4: Transformations, and circumference and conic sections.

## VI. METHODOLOGY

The course takes place in theory and practice sessions. In theory sessions, faculty presents the concepts, methods and principles. In practice sessions, students, under the guidance of faculty, solve diverse problems on matrices, analytic geometry, conics and so on. Student active participation is promoted.

## VII. GRADING FORMULA

The Final Grade PF is calculated as follow:

$$PF = (EP + EF + PP) / 3$$

EP: Mid-Term Exam

EF: Final Exam

PP: Average of 4 Practical Works

## VIII. BIBLIOGRAPHY

1. KOLMAN, BERNARD.  
Linear Algebra, Pearson Ed., 2016
2. GORDON FULLER, DALTON TARWATER.  
Analytic Geometry, Pearson Ed., 2015