



**NATIONAL UNIVERSITY OF ENGINEERING**  
**COLLEGE OF SCIENCES**  
**MATHEMATICS PROGRAM**

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**CM342 – SURFACES THEORY**

**I. GENERAL INFORMATION**

<b>CODE</b>	: CM342 Surfaces Theory
<b>SEMESTER</b>	: 6
<b>CREDITS</b>	: 5
<b>HOURS PER WEEK</b>	: 6 (Theory – Practice)
<b>PREREQUISITES</b>	: CM313 Multivariable Real Functions I CM314 Introduction to Topology
<b>CONDITION</b>	: Mandatory

**II. COURSE DESCRIPTION**

To give the student advanced knowledge about curves and surfaces in  $\mathbb{R}^3$ . To know the local and global characteristics about the theory of surfaces.

**III. LEARNING UNITS**

**1. Local Theory of Curves in Space  $\mathbb{R}^3$**

Curve Parameterization / Reparametrization / Arc length / Unitary speed curves / Tangent straight line to a curve / Tangent unit vector, normal vector, binormal vector / Curvature and torsion / Frenet differential system / Formulas to calculate curvature and torsion / Examples. Contact / Osculating plane / Osculating circle / Examples. Isometries and rigid movements in  $\mathbb{R}^3$ . Curvature Invariance and torsion by rigid movements in  $\mathbb{R}^3$  / Fundamental theorem of local theory of curves.

**2. Local Surface Theory in  $\mathbb{R}^3$**

Functionality differentiability  $F: U \rightarrow \mathbb{R}^m$ , where  $U$  is an open of  $\mathbb{R}^n$ . Jacobian matrix of  $F$ . Range of  $F$  in a point / Immersion, submersion, examples / Theorem of implicit functions, examples and applications / Local parametrizing. Definition of surface in  $\mathbb{R}^3$ , examples / Inverse Functions Theorem / Coordinate changes are diffeomorphisms between plane opens / Most common examples / Vectors tangent to a surface / Space tangent to a surface. Differential functions between two surfaces / Tangent application / Orientation. Adjustable surfaces / Examples. Möbius strip, non-orientable surface contained in  $\mathbb{R}^3$ . First fundamental form / Length of curves on surfaces / Angle between two curves / Area of a bounded region in a surface / Area of a spherical spindle / Area of a spherical triangle / Local isometries between surfaces / Examples. Second fundamental form / Normal curvature, geodetic curvature / Asymptotic lines and

geodesic lines on a surface / Examples. Geodesics on a surface of revolution, on a cylinder / Geodesic on the sphere.

### **3. Gauss Application and Shape Operator**

The shape operator is a self-adjoint transformation / Main curves and directions / Gaussian curvature. Average curvature / Classification of the points of a surface. Elliptical points, hyperbolic points, parabolic points, flat points / Surfaces of revolution of constant Gaussian curvature / Gaussian curvature and average curvature formulas as a function of the coefficients of the first and second fundamental forms / Gauss's Theorem. Gauss-Weingarten Equations / Codazzi-Mainardi Equations / Fundamental Theorem of Surface Theory.

## **IV. BIBLIOGRAPHY**

- Carmo, M. Do, Differentiable Curves and Surface.
- Eisenhart, Differential Geometry.
- O'Neil, Barrett, Elementary Differential Geometry.
- Struik, Geometría Diferencial Clásica.
- Willmore, An Introduction to Differential Geometry.