

# NATIONAL UNIVERSITY OF ENGINEERING COLLEGE OF SCIENCES

## **MATHEMATICS PROGRAM**

#### CM312 – ANALYSIS OF COMPLEX-VALUED FUNCTIONS

## I. GENERAL INFORMATION

**CODE** : CM312 Analysis of Complex-Valued Functions

SEMESTER : 6 CREDITS : 5

**HOURS PER WEEK** : 6 (Theory – Practice)

PREREQUISITES : CM314 Introduction to Topology

**CONDITION** : Mandatory

#### II. COURSE DESCRIPTION

To develop in a rigorous way the theory of complex functions with complex variable, highlighting those parts of the theory related to the applications of the topic. Provide an introduction to the application of residue and conformal transformation, which are used in the solutions of boundary problems. At the end of the course the student must also know acceptably the management of analytical functions and complex integration.

## **III. LEARNING UNITS**

#### 1. Analytical functions

Complex numbers and their properties / Holomorphic functions / Cauchy-Riemann equations. Powers series / Derivative of powers series and their properties / Exponential function. Sine and cosine trigonometric functions / Logarithms. Analytic functions of a variable.

## 2. Analytical functions on Convex Sets

Integration on arcs / Index of a curve and its properties / Cauchy local theorem / Cauchy's theorem for a triangle and for a convex open / Cauchy formula in a convex set / Local development in power series of an analytic function / Liouville's theorem / Fundamental theorem of algebra.

#### 3. Singularities

Singularities / Removable Singularities / Poles / Local Development / Essential Singularities.

# 4. Homological Cauchy Theorem

Cycles / Indexes on cycles / The generalized Cauchy theorem / Laurent series.

## 5. Waste Calculation

Meromorphic functions. Residues / The residue theorem

## 6. Homotopic Cauchy's Theorem

Homotopies / simply connected regions / The homotopic Cauchy theorem / Logarithms of functions on simply related regions / Calculation of zeros and poles of a meromorphic function. The theorem of Rouché

## 7. The Argument Principle

Continuous variation of the logarithm of a function along a path / Continuous variation of the argument The argument principle.

## 8. The Mapping Theorem

Arzelá-Ascoli's theorem. Normal families / Conformal mapping theorem.

#### 9. Infinite Products

Infinite products / Convergence / Hurwitz's theorem and its consequences / Structure of the zeros of an infinite product / Weierstras's theorem on sets without accumulation point / Applications.

## IV. BIBLIOGRAPHY

- Churchil, Ruel, Variable compleja y Aplicaciones.
- Derrick, William, Variable Compleja con Aplicaciones.
- Flanigan, Complex Variables: Harmonic and Analytic Function.
- Markushevich, Teoría de Funciones Analíticas, Vol. I y II.
- Serge, Lang, Complex Analysis.
- Hauser, Arthur, Variable Compleja.
- Ahlfors, L., Análisis de Variable Compleja.
- Cartan, Henri, Elementary Theory of Analytic Functions of One or Several Complex Variables.
- Neto, Alcides Lins, Funções de uma Variável Complexa.
- Rudin, Walter, Real and Complex Analysis.