



**NATIONAL UNIVERSITY OF ENGINEERING**  
**COLLEGE OF INDUSTRIAL AND SYSTEMS ENGINEERING**  
**SYSTEMS ENGINEERING PROGRAM**

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**SYLLABUS - CB132 MULTIVARIABLE CALCULUS**

**I. GENERAL INFORMATION**

<b>CODE</b>	: CB132
<b>SEMESTER</b>	: 3
<b>CREDITS</b>	: 5
<b>HOURS PER WEEK</b>	: 6 (Theory – Practice)
<b>PREREQUISITES</b>	: CB111 Linear Algebra, CB131 Integral Calculus
<b>CONDITION</b>	: Compulsory
<b>INSTRUCTOR</b>	: Jose Alvarado, Javier Echeandia, Osmar Bermeo

**II. COURSE DESCRIPTION**

This theoretical and practical course has the objective of providing students with the necessary criteria and methods to model, analyze, and solve physical, geometric and optimization problems. The content of this course is divided in five learning units and encompasses the following subjects: Vectorial functions, curves, function of several variables, directional and partial derivatives, double and triple integral, line and surface integral, Stokes' theorem and Gauss' theorem, first and second order differential equations.

**III. COURSE OUTCOMES**

1. Acknowledge the scientific nature of mathematics and assess its rigor and objectivity.
2. Analyze fundamental theorems of mathematics and rigorously apply them to situations with specific problems.
3. Interpret the concept of Vectorial function and apply it to calculate limits, derivatives and integrals and assess its importance in the solution to engineering problems.
4. Understand the concept of real function of several variables and apply it to calculate limits, partial derivatives, double and triple integrals.
5. Define and calculate line, divergence and rotational integrals and assess its importance in the solution to engineering problems.
6. Identify the type, order and degree of an ordinary differential equation and apply solution methods in each case.

**IV. LEARNING UNITS**

**1. VECTORIAL FUNCTIONS / 10 HOURS**

Vectorial functions: Limits, continuity, derivatives, integrals, arc length / Fundamental vectors: tangent, Normal and binormal vectors / Fundamental planes: osculating, normal and rectifying plane.

**2. FUNCTIONS OF SEVERAL VARIABLES / 15 HOURS**

Real function of several variables: limits, continuity. Partial derivate / Directional and gradient derivates, tangent plane. Chain rule, implicit derivate / Second derivate criterion. Maxima and minima / Lagrange multiplier applied to conditioned problems / Double integrals: definition, change of variable in double integrals. Double integral in polar coordinate systems.

### **3. DOUBLE AND TRIPLE INTEGRALS / 15 HOURS**

Double integrals: definition, change of variable in double integrals. Double integral in polar coordinates system / Center of mass / Moments of inertia / Triple integral – volume / Change of variables in triple integrals.

### **4. LINE INTEGRALS / 15 HOURS**

Line integrals. Path-independent line integral. Work. Green's theorem / Surface area. Surface integral of the curl / Divergence of a vector field. Flow of vector fields / Stokes' theorem. Gauss' theorem. Applications.

### **5. ORDINARY DIFFERENTIAL EQUATIONS / 15 HOURS**

Ordinary differential equations: order and degree / Equation with separable variables, equations with exact variables / Linear, homogeneous and Bernoulli differential equations / Application: Exponential growth and decay / Second order differential equations with homogeneous and non homogeneous constants / method of undetermined coefficients. Application: Orthogonal trajectories.

## **VI. METHODOLOGY**

This course is carried out through theory and practical sessions. In theory sessions, the instructor introduces concepts, theorems and applications. In practical sessions, several problems are solved and their solution is analyzed. In all sessions, students' active participation is encouraged.

## **VII. EVALUATION FORMULA**

The average degree PF is calculated as follows:

$$PF = (EP + EF + (P1+P2+P3+P4)/3) / 3$$

EP: Mid-Term Exam

EF: Final Exam

P#: Quizzes

## **VIII. BIBLIOGRAPHY**

1. **STEWART, JAMES**  
Multivariate calculus (Spanish)  
I.T.E. Editorial, (2005)
2. **APOSTOL, TOM**  
Calculus Volume II  
Reverte Editorial (2008)
3. **DEMIDOVICH**  
5000 Problems of Mathematical Analysis  
Paraninfo Editorial (2008)